AN ACCURATE, FIELD MATCHING ANALYSIS OF WAVEGUIDES OF COMPLEX CROSS–SECTIONAL GEOMETRY LOADED WITH MAGNETIZED FERRITE RODS.

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ABSTRACT

In this contribution we present developments to our technique of analysis of complex waveguides [1]. We may now analyze waveguides having complicated cross–sectional geometry and comprising circularly–cylindrical ferrite–dielectric rods magnetized in a longitudinal direction. This class of waveguides includes rectangular waveguides loaded with ferrite rods, which have not, to the best knowledge of authors, been rigorously analyzed yet, despite their wide spread usage in devices. We also present experimental data validating our theory, and examples of potentially useful structures which can be treated by our method (e.g. ferrite loaded waveguides of crossed-rectangular and finned – circular cross sections). The inner products encountered in the method are computed via FFT which resulted in 3–fold increase of computational speed and 2–fold reduction of computer storage.

INTRODUCTION

In this contribution we present extensions to our technique of analysis of complex waveguides, developed by us [1] from a method of analysis of planar circuits[2, 3, 4]. We may now analyze waveguides having complicated cross-sectional geometry and comprising circularly–cylindrical ferrite–dielectric rods magnetized in a longitudinal direction. This class of waveguides includes rectangular waveguides loaded with ferrite rods, which have not, to the best knowledge of authors, been rigorously analyzed yet. We also present experimental data validating our theory. The examples of new, potentially useful structures (e.g. ferrite loaded waveguides of crossed-rectangular and finned – circular cross sections) are given.

The analytical technique we used is based on the field–matching method and follows the approach applied by us to the analysis of complex waveguides loaded with dielectric materials [1]. Although the field matching is not usually recognized as a very flexible method (as opposed to numerical methods e.g. FDTD), the formulation we use let us analyze a very wide class of waveguides (see figs 1 and 3).

OUTLINE OF THE ANALYSIS

Consider a waveguide of a cross–sectional geometry sketched in fig. 2. In brief the analysis consists of the following steps:

1. We divide the structure into regions in which solutions to Maxwell equations can be found relatively easy, namely:
   - Inner cylindrical guide (IG) defined in cylindrical coordinates (delineated with curve ζ – fig. 1). IG is transversely inhomogeneous. It is composed of an arbitrary number of cylindrical layers of which all but the external most may be made of longitudinally magnetized ferrite material.
   - Isotropic and homogeneous attached guides (AG). These guides may have a variety of shapes, an example being a rectangular guide open at one side (as in fig.1), or a sectoral guide (fig.3)
2. We find the field in AGs in terms of series of eigenfunctions fulfilling all the boundary conditions except of those on the interface with IG.
3. We find the general solution of Maxwell eq. in IG. Here we exploit the *Transfer Matrix* concept (modified for the case of ferrite IG) to treat the multilayered structure of IG [1], and express the field quantities in the external layer of IG in terms of the amplitudes of the innermost.
4. Now we match the fields, i.e. match the general solutions found in distinguished regions. A point to note here is that IG and AGs regions partially overlap (fig.1), and therefore one can delineated a number of interfaces on which the continuity conditions for the e.m fields can be formulated. The practical importance have however only such interfaces on which one can take advantage of the orthogonality properties of eigenfunctions i.e. ξ and ζ. In the field matching process we apply a bi-directional orthogonal expansion

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and thus use both interfaces. Continuity conditions for $\vec{E}$ fields are formulated on ζ:

$$\left.\begin{array}{rcl} E_x^\circ & = & \sum_{i=1}^{M} E_x^\perp \mathcal{B}_i(\varphi) \\ \sum_{i=1}^{M} \vec{E}^\circ \vec{a}_{z_i} \mathcal{B}_i(\varphi) & = & \sum_{i=1}^{M} E_{z_i}^\perp \mathcal{B}_i(\varphi) \end{array}\right|_\zeta$$

where: superscripts $^\circ$ and $^\perp$ represents field from IG and AG respectively, subscript $_i$ denote i–th AG guide, and $\mathcal{B}$ is a function selecting i–th AG region. $\vec{H}$ fields we match on ξ:

$$\left.\begin{array}{rcl} H_x^\circ & = & \sum_{i=1}^{M} H_x^\perp \mathcal{B}_i(\varphi) \\ H_\varphi^\circ & = & \sum_{i=1}^{M} H^\perp{}_\varphi \mathcal{B}_i(\varphi) \end{array}\right|_\xi$$

5. We transform these set of equations into a matrix algebraic system taking the inner products of both side of the above equations with eigenfunctions of AG for the $\vec{E}$ fields and with eigenfunctions of IG for $\vec{H}$ fields. The resulting matrices we manipulate so that the unknown amplitudes of eigenfunctions in IG are eliminated. Finally, we arrive at the dispersion equation, demanding the determinant of the homogeneous algebraic system obtained to be equal to zero. It is worth noticing that bi-directional orthogonalization approach used, ensures better numerical behavior of the method (as opposed to unidirectional in [2, 3, 4]) since the number of modes taken into the e.m. approximation in AG must not necessarily equal to that of IG, providing greater flexibility of the algorithm.

APPLICATION OF FFT

Critical from the point of view of the efficiency of the computations is the evaluation of the inner products. We have noticed that the inner products encountered in our algorithm have the form:

$$S_k = \int_{-\theta_1}^{\theta_1} \mathcal{S}(\varphi) e^{-jk\varphi}\, \mathrm{d}\varphi$$

where S is a certain function of φ and the products must be computed for k ranging from 1 to K, where K is number of eigenfunctions taken into account in the IG. Simple transformation of variables, and a modification of S:

$$\overline{\mathcal{S}}(\varphi) = \begin{cases} 0 & |\varphi| > \theta_1 \\ \mathcal{S} & |\varphi| \le \theta_1 \end{cases} \qquad \overline{\varphi} = \varphi + \pi$$

leads to the modified formula for S_k:

$$S_k = 2\pi(-1)^k \frac{1}{2\pi} \int_0^{2\pi} \overline{\mathcal{S}}(\overline{\varphi}) e^{-jk\overline{\varphi}}\, \mathrm{d}\overline{\varphi}$$

In this formula we easily recognize a term which represents a Fourier expansion coefficient of function $\overline{\mathcal{S}}(\varphi)$ and which may be efficiently computed via FFT. Using this strategy we have gained 3–fold increase of computational speed and 2–fold reduction of computer storage.

VERIFICATION AND RESULTS

Numerous numerical and experimental tests have been carried out in order to verify the method. We shell now briefly present few examples. A section of a rectangular waveguide loaded with longitudinally magnetized ferrite rod was terminated with metallic walls to form a resonator. The resonance frequency was measured as a function of the applied external biasing magnetic field – figs 4 & 5. The interaction between ferrite medium and e.m. wave weakens when the ferrite rod is shifted towards the narrow waveguide wall – fig. 4. Conversely, if the ferrite material is positioned close to the wider waveguide wall the interaction with e.m. field intensifies and, simultaneously, the sensitivity to the strength of the biasing magnetic field increases – fig. 5. The theoretical curves are compared with experimental data. A good agreement may be observed. Although this type of structure is widely known from practical application (e.g. in phase shifters) it has not been rigorously analyzed yet.

In a cross–rectangular waveguide comprising longitudinally magnetized ferrite rod, a Faraday rotation phenomenon may be observed. This structure provides however substantial flexibility in shaping the frequency characteristics. Fig. 6 presents some examples.

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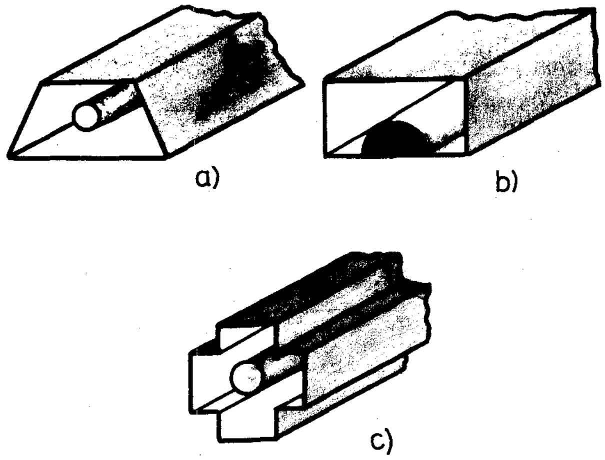

Figure 1: Examples of ferrite or dielectric loaded structures which can be treated by our method. *a*– trapezoidal waveguide, *b*– "image" guide, *c*– cross–rectangular guide. AGs assumed as rectangular guides. Other examples in fig.3

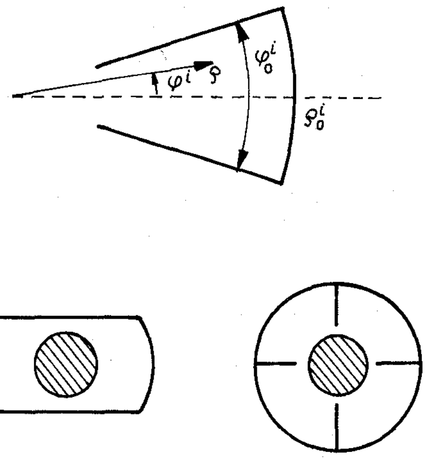

Figure 3: AG as sectoral guides – *a*), and examples of waveguides with sectoral AGs — *b*) and *c*).

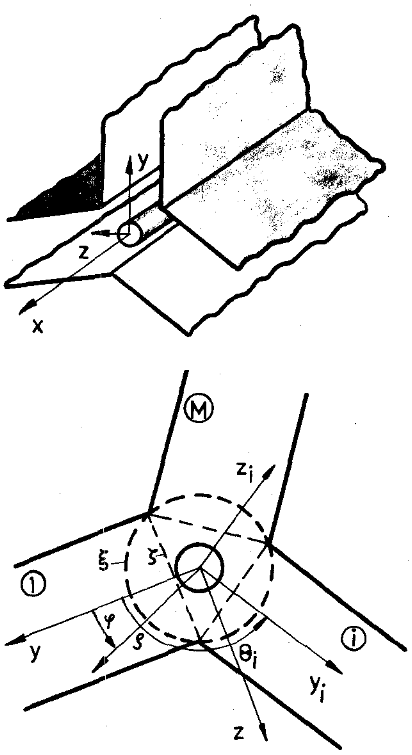

Figure 2: Structure under analysis – schematic presentation *a* –3D, *b* – cross-sectional view.

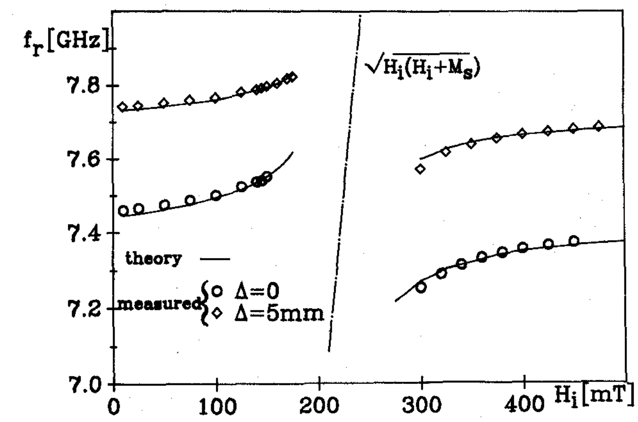

Figure 4: Resonance frequency versus external dc magnetic field in resonator build from a section of rectangular guide loaded with a ferrite rod. (ϕ3mm, M_s=950$\frac{1}{4\pi}\frac{kA}{m}$, ϵ_f=13), a=22.86mm, b=10.16mm. Δ_h – offset of the rod towards narrower waveguide wall.

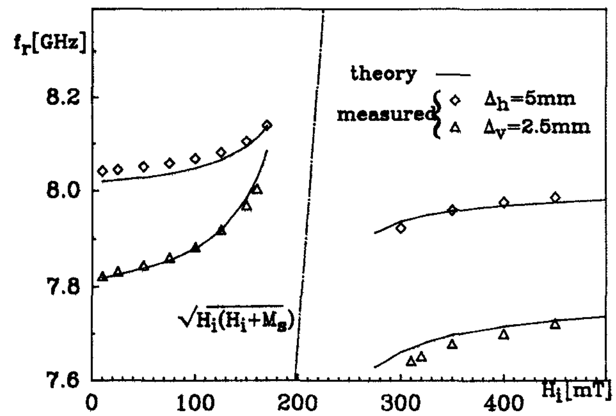

Figure 5: Resonance frequency versus external dc magnetic field in resonator build from a section of rectangular guide loaded with a ferrite rod. (ϕ3mm, M_s=1750$\frac{1}{4\pi}\frac{kA}{m}$, ϵ_f=13.5), a=22.86mm, b=10.16mm. Δ_h, Δ_v - offset of the rod towards narrower or wider waveguide wall respectivly.

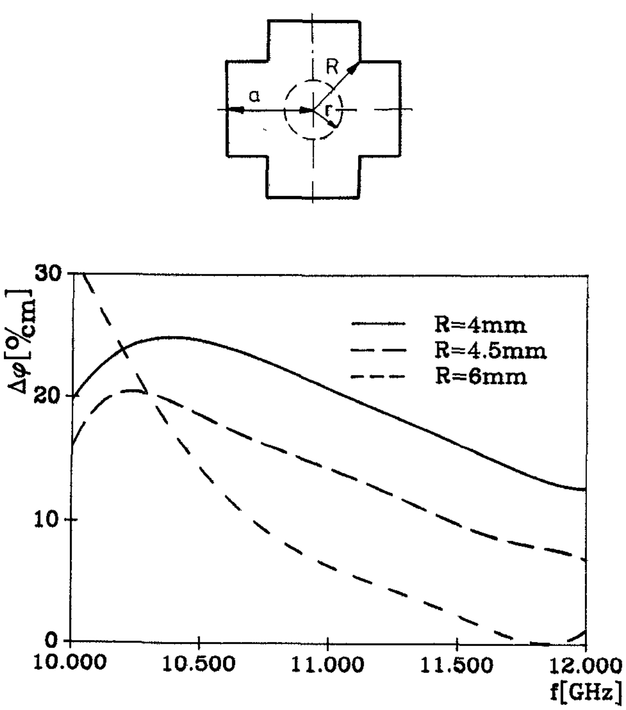

Figure 6: Rotation of the polarization plane in a cross-rectangular waveguide comprising longitudinally magnetized ferrite. r=2.5mm, a=7mm, M_s=2200$\frac{1}{4\pi}\frac{kA}{m}$, H_i=0, ϵ_f=13.5.